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SIGNALS AND SYSTEMS

KUESTION

Signals and Systems

Contents

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Manual for Kuestion

Why Kuestion?

It's very overwhelming for a student to even think about finishing 100-200 questions per chapter when the clock is ticking at the last moment. This is the reason why Kuestion serves the purpose of being the bare minimum set of questions to be solved from each chapter during revision.

What is Kuestion?

A set of 40 questions or less for each chapter covering almost every type which has been previously asked in GATE. Along with the Solved examples to refer from, a student can try similar unsolved questions to improve his/her problem solving skills.

When do I start using Kuestion?

It is recommended to use Kuestion as soon as you feel confident in any particular chapter. Although it will really help a student if he/she will start making use of Kuestion in the last 2 months before GATE Exam (November end onwards).

How do I use Kuestion?

Kuestion should be used as a tool to improve your speed and accuracy chapter wise. It should be treated as a supplement to our K-Notes and should be attempted once you are comfortable with the understanding and basic problem solving ability of the chapter. You should refer K-Notes Theory before solving any "Type" problems from Kuestion.

Type 1: Delta Function

For Concept refer to Signals and Systems K-Notes, Signals

Common Mistake:

Divide by the coefficient of "t" in delta function while integrating.

Sample Problem 1:

What is the value of $\int\limits_{-\infty}^{\infty}(\delta(t)\cos 2t + \delta(t-2)\sin 2t)dt$

(A)1+cos4   (B)cos4   (C)1+sin4   (D)sin4

Solution: (C) is correct option

$$\int\limits_{-\infty}^{\infty}(\delta(t)\cos 2t + \delta(t-2)\sin 2t)dt = \cos(0) + \sin(2*2) = 1+\sin 4$$

Unsolved Problems:

Q.1 $\int\limits_{-3}^{6}(t^2+2t+2)\delta(2t+3)dt$ =?

(A) 4.6   (B) 3.6   (C) 17   (D) None of the above

Q.2 Find the Fourier transform of the signal $x(t)=\delta(t+1)\times\delta(t-1)$

(A) $\dfrac{2}{1+j\omega}$ (B) $\dfrac{2}{1-j\omega}$ (C)2cosω   (D)None

Q.3 The value $\int\limits_{-2}^{2}([t-3]\delta(2t+2)+8\cos\pi t+\delta'(t-0.5))dt$ is

(A)23.13   (B)13.56   (C)6.39   (D) 7.85

Q.4 The value $\int\limits_{-\infty}^{0}4\delta(2t-2)\sin 2t)dt$ is

(A)4sin4   (B)2sin4   (C)sin2   (D)0

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Type 2: Signal Period

For Concept, refer to Signals and Systems K-Notes, Signals

Common Mistake:

If one signal period is rational and other is irrational, then sum of the two signals can never be periodic.

Sample Problem 2:

The time period 'T' of the signal $x(t)=3\sin\left(\dfrac{t}{2}\right)+4\cos(4t)+8\sin(t)$ is equal to

(A) π (B) 3π (C) 4π (D) $\dfrac{\pi}{3}$

Solution: (C) is correct option

Period of $3\sin\left(\dfrac{t}{2}\right)$ => $T_1 = \dfrac{2\pi}{0.5} = 4\pi$

Period of $4\cos(4t)$ => $T_2 = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

Period of $8\sin(t)$ => $T_3 = \dfrac{2\pi}{1} = 2\pi$

Now , Period of $x(t)$ => $T = L.C.M.\{T_1, T_2, T_3\}$

$$= L.C.M.\left\{4\pi, \dfrac{\pi}{2}, 2\pi\right\}$$

$$= 4\pi$$

Unsolved Problems:

Q.1 The period of the signal x(t)=8Sin(0.8πt+π/4) is

(A)0.4π sec (B)0.8π sec (C)1.25 sec (D)2.5 sec

Q.2. Let x(n) be a discrete – time signal, and Let $y_1(n) = x(2n)$ &

$$y_2(n) = \begin{cases} x[n/2] \ , \ n \text{ even} \\ \qquad 0 \ , \ n \text{ odd} \end{cases}$$

Consider the following statements

(1) If x(n) is periodic, then $y_1(n)$ is periodic

(2) If $y_1(n)$ is periodic, then x(n) is periodic

(3) If x(n) is periodic, then $y_2(n)$ is periodic

 Which of the statements are TRUE?

 (A) 1, 2, 3   (B) 1, 3   (C) 1, 2   (D) 2, 3

Q.3 A discrete –time signal is given as $x[n] = \cos\left(\dfrac{n}{8}\right)\cos\left(\dfrac{\pi n}{8}\right)$. The signal is

(A) Periodic with period 16π

(B) Periodic with period $16(\pi+1)$

(C) Periodic with period 8

(D) Not Periodic

Q.4 The period of the signal $6e^{j\frac{4\pi}{3}t} + 8e^{j(3\pi t+\frac{\pi}{2})}$ is

(A)4   (B)5   (C)6   (D)7

Q.5 The period of the signal $s(t) = \sin\dfrac{5t}{2} + 3\cos\dfrac{6t}{2} + 3\sin(\dfrac{t}{7}+30^0)$ is ?

(A)140π (B) 120π (C) 160π (D)Not periodic
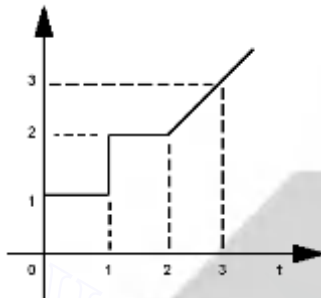
Type 3: Signal Representation

For Concept, refer to Signals and Systems K-Notes, Signals.

Common Mistake:

Make sure that same argument appears in function as the unit step function multiplied to it

before taking any Transform. eg. The term with u(t-2) should be (t-2) as in sample problem.

Sample Problem 3:

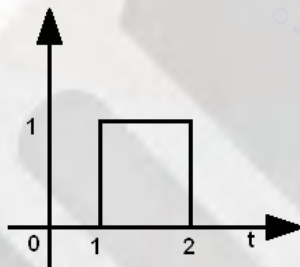Which of the following gives correct description of the wave form.

(A) u(t)+u(t-1)

(B) u(t)+(t-1)u(t-1)

(C) u(t)+u(t-1) +(t-2)u(t-2)

(D) u(t)+(t-2)u(t-2)

Solution: (C) is correct option

u(t-1)-u(t-2)

Similarly in this question

For 0≤t<1 $x_1(t)$=u(t)-u(t-1)

For 1≤t<2 $x_2(t)$=2[u(t-1)-u(t-2)]

For 2≤t $x_3(t)$=tu(t-2)=(t-2)u(t-2)+2u(t-2)

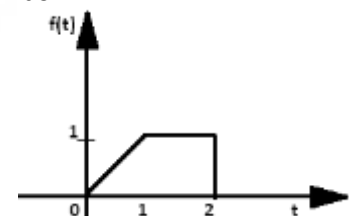Now, x(t)= $x_1(t)$+$x_2(t)$+ $x_3(t)$= u(t)+u(t-1) +(t-2)u(t-2)

Unsolved Problems:

Q.1 The function f(t) shown in the figure will have Laplace transform as

(A) $\dfrac{1}{s^2} - \dfrac{1}{s}e^{-s} - \dfrac{1}{s^2}e^{-2s}$
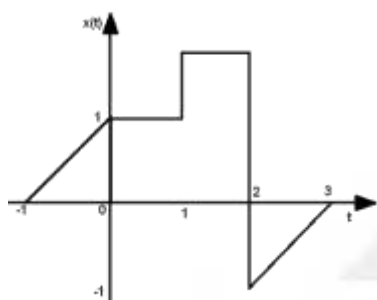
(B) $\dfrac{1}{s^2}\left(1 - e^{-s} - e^{-2s}\right)$

(C) $\dfrac{1}{s}\left(1 - e^{-s} - e^{-2s}\right)$

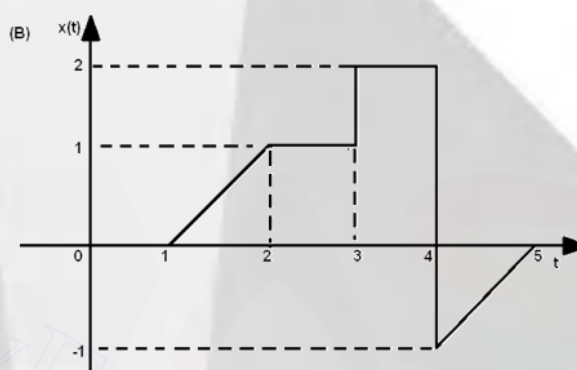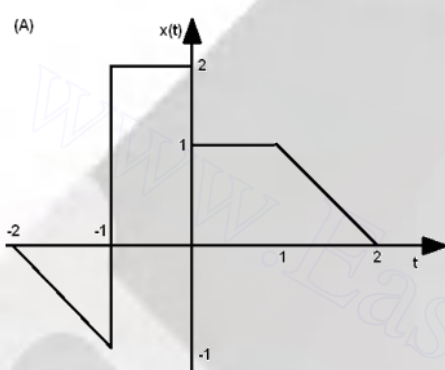(D) $\dfrac{1}{s^2}\left(1 - e^{-s} - se^{-2s}\right)$
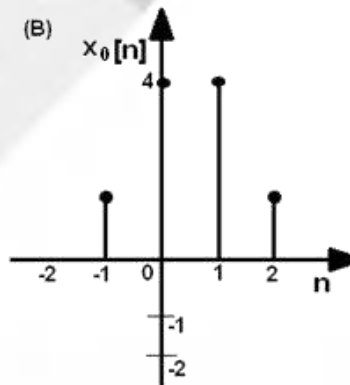
Q.2 If a plot of signal x(t) is as shown in the figure

Then the plot of the signal x(1-t) will be

(A)

(B)

(C)

(D)
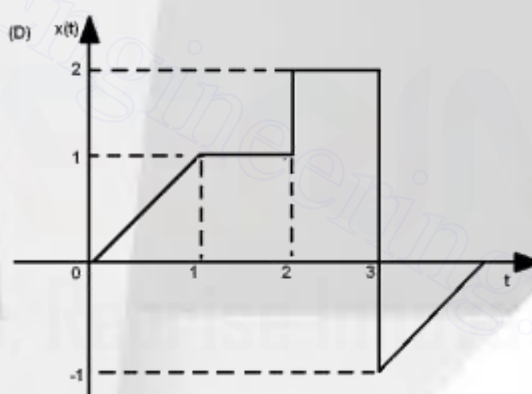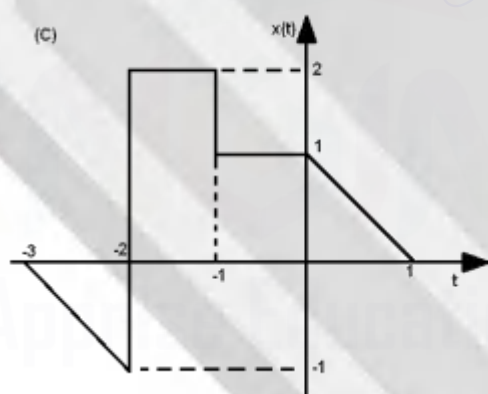
Q.3 The odd part of the sequence $x[n] = \{6, 4, 2, 2\}$ is

(A)

(B)

7

(C)

(D)
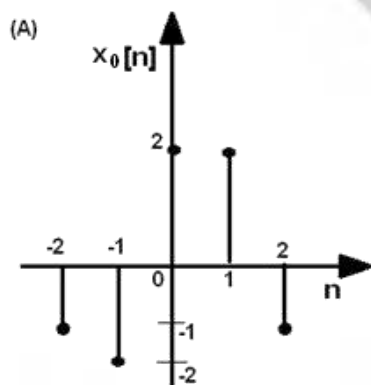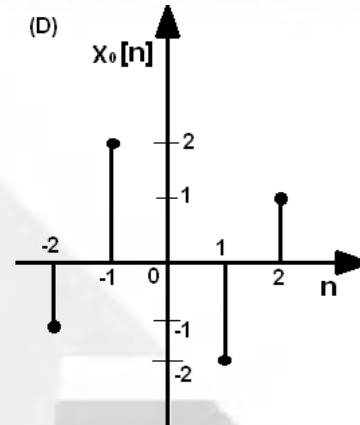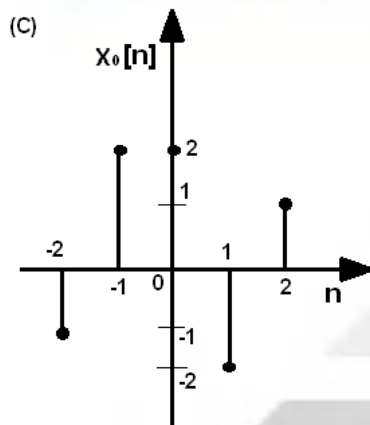
Q.4 Consider two signal x(t) and y(t) shown in figure and there Laplace transform pairs

$x(t)\xrightarrow{LT}X(s)$ and $y(t)\xrightarrow{LT}Y(s)$
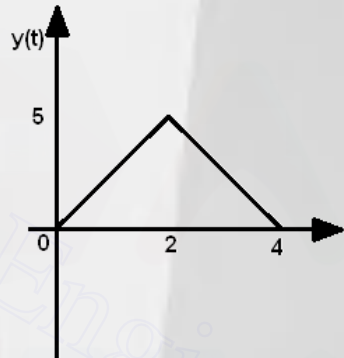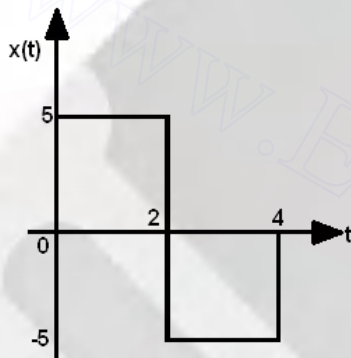
If $X(s)=\dfrac{1}{s}\left[5-10e^{-2s}+5e^{-4s}\right]$. Then Y(s) will be?

(A) $5-10e^{-2s}+5e^{-4s}$

(B) $\dfrac{1}{2s^2}\left[5-10e^{-2s}+5e^{-4s}\right]$

(C) $s\left[5-10e^{-2s}+5e^{-4s}\right]$

(D) $\dfrac{1}{s^2}\left[5-10e^{-2s}+5e^{-4s}\right]$

Q.5 The input output pair for a LTI system is given below.
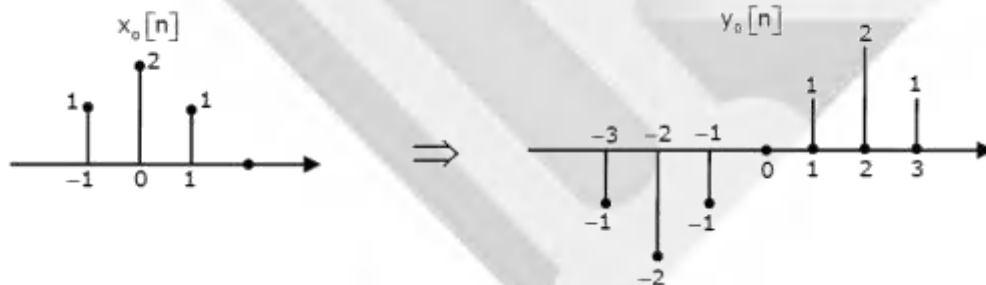
Find the output y[n] if input is as shown below.

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(A) $y_0[n] + y_0[n-1] + y_0[n-3]$

(B) $y_0[n-1] + y_0[n+1] + y_0[n]$

(C) $y_0[n-1] + y_0[n-3] + y_0[n-5]$

(D) $y_0[n] + y_0[n-1] + y_0[n-2]$

Type 4: Signal RMS Value

Sample Problem 4:

The rms value of the periodic waveform given in figure is
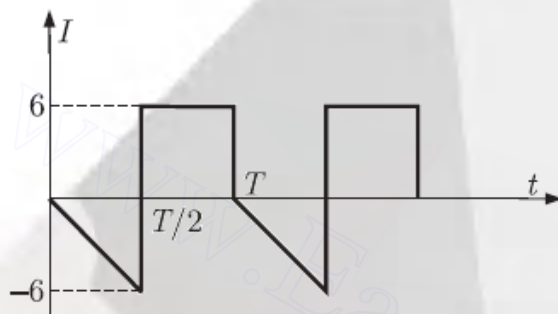
(A) $2\sqrt{6}A$

B) $6\sqrt{2}A$

(C) $\sqrt{\dfrac{4}{3}}A$

(D) 1.5 A

Solution: (A) is correct option

Given $\quad I(t) = \begin{cases} -\dfrac{12}{T}t, & 0 < t < \dfrac{T}{2} \\[2mm] 6, & \dfrac{T}{2} < t < T \end{cases}$

$$\frac{1}{T}\int_0^T I^2 dt = \frac{1}{T}\left[\int_0^{\frac{T}{2}} \left(\frac{-12t}{T}\right)^2 dt + \int_{\frac{T}{2}}^T 6^2 dt \right]$$

$$\frac{1}{T}\int_0^T I^2 dt = \frac{1}{T}\left[\frac{144}{T^2} \int_0^{\frac{T}{2}} t^2 dt + \frac{36T}{2} \right]$$

$$\frac{1}{T}\int_0^T I^2 dt = \frac{1}{T}\left[\frac{144}{T^2} \times \frac{T^3}{8} + 18T \right]$$

$$= 6 + 18 = 24$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I^2 dt} = \sqrt{24} = 2\sqrt{6}A$$

Unsolved Problems:

Q.1 If current of $-6\sqrt{2}\sin(100\pi t) + 6\sqrt{2}\cos(300\pi t + \frac{\pi}{4}) + 6\sqrt{2}$ A passed through a true RMS ammeter, the meter reading will be

(A) $6\sqrt{2}$A   (B) $\sqrt{126}$ A   (C) 12 A   (D) $\sqrt{216}$ A

Q.2 The average value of the periodic signal x(t) shown in figure is

(A) 5/6   (B) 1   (C) 5   (D) 6

Q.3 Find the R.M.S. value of the function $f(t) = \sin\omega t + \cos\omega t$ across $[0,1]$

(A) $\sqrt{1 + \dfrac{\sin^2\omega}{\omega}}$ (B) $\sqrt{1 + \dfrac{\cos^2\omega}{\omega}}$

(C) $\sqrt{1 + \dfrac{\sin^2\omega}{2\omega}}$ (D) $\sqrt{1 + \dfrac{\cos^2\omega}{2\omega}}$

Q.4 Find the Average value of the function $f(t) = \sqrt{t+1}$ across $[0,3]$

(A) 0   (B) $\dfrac{7}{18}$ (C) $\dfrac{7}{9}$ (D) $\dfrac{14}{9}$

Type 5: Energy and Power Signals

For Concept, refer to Signals and Systems K-Notes, Signals

Common Mistake:

A periodic signal is always a Power Signal and a finite signal is always an Energy Signal.

Sample Problem 5:

The power in the signal $s(t) = 8\cos(20\pi t - \frac{\pi}{2}) + 4\sin(15\pi t)$ is?

(A) 40   (B) 41   (C) 42   (D) 82

Solution: (A) is correct option

$$s(t) = 8\cos(20\pi t - \frac{\pi}{2}) + 4\sin(15\pi t)$$

$$s(t) = 8\sin(20\pi t) + 4\sin(15\pi t)$$

$$\text{Power} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |s(t)|^2 \, dt$$

$$\text{Power} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |8\sin(20\pi t) + 4\sin(15\pi t)|^2 \, dt$$

$$\text{Power} = \frac{8^2}{2} + \frac{4^2}{2} = 40$$

Unsolved Problems:

Q.1 The frequency spectra of a x(t) is given below.
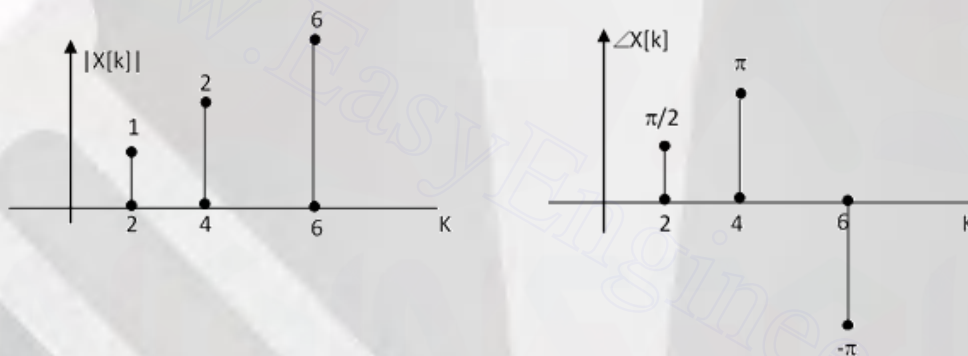
The power of x(t) is

(A) 41w   (B) 20.5w   (C) 24w   (D) 25w

Q.2 Consider a continuous time signal $x(t) = \delta(t+2) - \delta(t-2)$. The value of E_∞ for the signal

$$\int_{-\infty}^{t} x(\tau) d\tau = ?$$

(A)2J   (B)4J   (C)0.5J   (D)0.25J

Q.3 The power contained in the first 2 harmonics of periodic signal shown in figure below

(A)0.3083 W   (B)0.33 W   (C)0.67 W   (D)0.0308 W
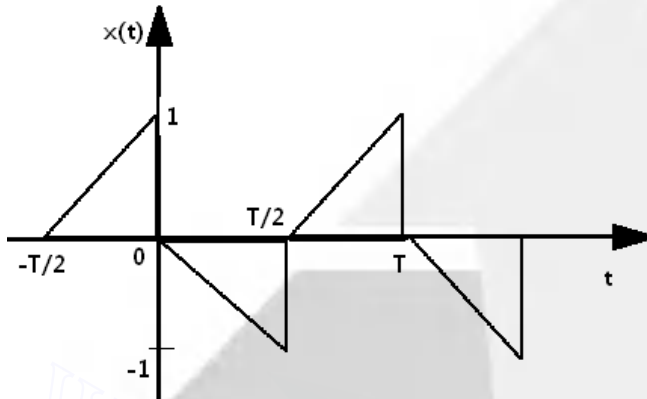
Q.4 Figure below shows the P.S.D. of a power signal x(t). Then the average power of the signal is?
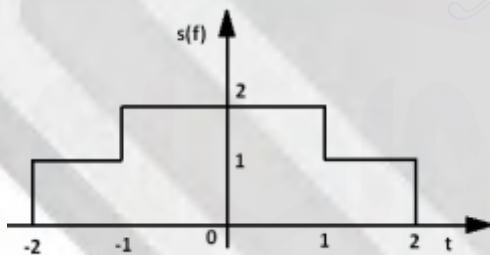
(A)1 W   (B)2 W   (C)3 W   (D)6 W

Q.5 An ideal second order low pass filter shown below with cut off frequency of 1 rad/sec is supplied with x(t)=e⁻ᵗu(t). Energy at response of the system
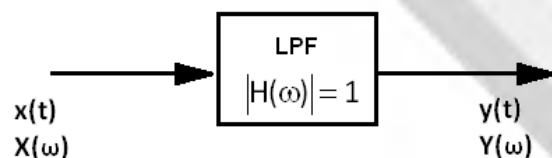
(A) 1 J   (B) $\dfrac{1}{2}$ J   (C) $\dfrac{1}{4}$ J   (D) $\dfrac{1}{8}$ J

Q.6 The discrete time signal is given as

$$f[n] = \cos\left(\frac{\pi n}{3}\right) \times (u[n] - u[n-6]).$$ The energy of the signal is ?

(A)1   (B)2   (C)3   (D)4

Type 6: LTI System Properties

For Concepts, refer to Signals and Systems K-Notes, LTI Systems

Common Tip:
An LTI System by default is an Invertible System.

Sample Problem 6:

The input x(t) and output y(t) of a system are related as $y(t) = \int\limits_{-\infty}^{t} x(\tau)\cos(3\tau)d\tau$ The system is.

(A) time-invariant and stable   (B) stable and not time-invariant
(C) time-invariant and not stable   (D) not time-invariant and not stable

Solution: (D) is correct option

$$X(t) \longrightarrow y(t) = y(t) = \int\limits_{-\infty}^{t} x(\tau)\cos(3\tau)d\tau$$

$$X(t-t_0) \longrightarrow o/p$$

$$= \int\limits_{-\infty}^{t} x(\tau - t_0)\cos(3\tau)d\tau, \ \text{Let } \tau - t_0 = \tau_1$$

$$= \int\limits_{-\infty}^{t-t_0} x(\tau_1)\cos(3t_0 + 3\tau_0)d\tau_1$$

$$y(t-t_0) = \int\limits_{-\infty}^{t-t_0} x(\tau)\cos(3\tau)d\tau$$

$$o/p \neq y(t-t_0)$$

∴The system is time-varying or not time-invariant.

For a bounded input , x(t)=cos(3t)u(t)

$$y(t) = \int_0^t \cos^2(3\tau)d\tau$$

$$= \int_0^t \frac{1}{2}d\tau + \int_0^t \frac{1}{2}\cos(6\tau)d\tau = I_1 + I_2$$

I_1 is unbounded, $I_1 \longrightarrow \infty$ as $t \longrightarrow \infty$

∴The system is not stable.

Unsolved Problems:

Q.1 The following difference equation represents input output relationship of a discrete system. $y(n) = 2y(n-1) - 3y(n-2) + 4x(n-1)$
The nature of the system is

(A) Causal, Time Variant and Unstable
(B) Causal, Time Invariant and Unstable
(C) Anticausal, Time Invariant and Unstable
(D) Non causal, Time variant, and stable

Q.2 The input and output relationship of a system is given below $\dfrac{d^2 y(t)}{dt^2} + \dfrac{d^2 x(t)}{dt^2} + 2tx(t) = 6$

(A) linear, time invariant, causal   (B) Non-linear, time variant, non-causal

(C) Non-linear, time invariant, causal   (D) linear, time variant, non-causal

Q.3 The impulse response of a discrete LTI system is given by h(n) = - (0.25)^{-n} u(n – 4).

The system is

(A) Causal & Stable   (B) Causal & Unstable

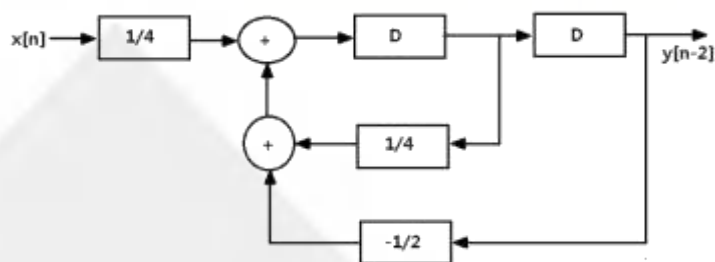(C) Non causal & Stable   (D) Non – Causal & Unstable

Q.4 The transfer function of a system is given by $H(z) = \dfrac{z(3z-2)}{z^2 - z - \dfrac{1}{4}}$ The system is

(A) Causal and stable

(B) Causal, stable and minimum phase
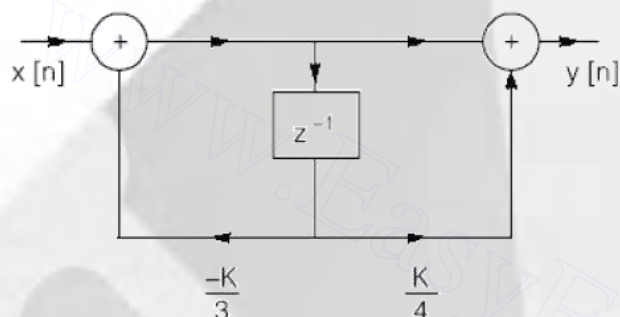
(C) Minimum phase

(D) None of the above

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Q.5 The system given below is

(A) Linear and Causal

(B) Non-Linear and Causal

(C) Linear and Non causal

(D) Non-Linear and Non – Causal

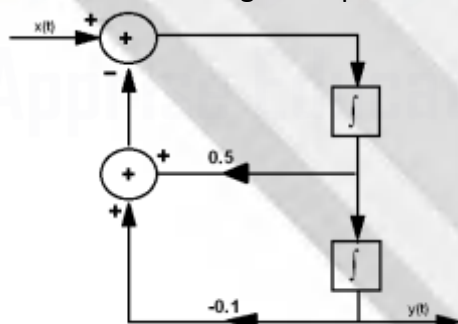**Q.6** Consider a digital filter defined by the following structure

The range of k for which the system is stable, is

(A) $|k| < 3$ (B) $|k| > 3$ (C) $|k| < \dfrac{1}{3}$ (D) $|k| > \dfrac{1}{3}$

Q.7 The block diagram representation of a CT system in the figure below . The system is

(A)BIBO Stable   (B)BIBO Unstable   (C)Marginally Stable   (D)None

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Type 7: Convolution

For Concept refer to Signals and Systems K-Notes, LTI Systems

Common Mistake:
This is a lengthy question, take care of calculations as this is the most probable error.

Sample Problem 7:
Given two continuous time signals $x(t)=e^{-t}$ and $y(t)=e^{-2t}$ which exist for t>0 , the convolution $z(t)=x(t)*y(t)$ is

(A) $e^{-t}- e^{-2t}$ (B) e^{-3t} (C) e^t (D) $e^{-t}+ e^{-2t}$

Solution: (A) is correct option

For $x(t)=e^{-t}$, $t>0 \xrightarrow{\text{L.T.}} X(s)=\dfrac{1}{s+1}$

For $y(t)=e^{-2t}$, $t>0 \xrightarrow{\text{L.T.}} y(s)=\dfrac{1}{s+2}$

$z(t)=x(t)*y(t),\quad Z(s)=x(s)\times Y(s)$

If $Z(s)=\dfrac{1}{(s+1)(s+2)}=\dfrac{1}{s+1}-\dfrac{1}{s+2}$

Taking inverse L.T.

$z(t)= (e^{-t}- e^{-2t})u(t)$

Unsolved Problems:

Q.1 The input of a system $x(t) = e^{-6t} u(t)$ and output $y(t) =\left(\dfrac{e^{-6t}-e^{-8t}}{2}\right)u(t)$

The step Response s(t) is

(A) $s(t) = \dfrac{1}{8}(1-e^{-8t})u(t)$

(B) $s(t) = \dfrac{1}{8}(e^{-8t})u(t)$

(C) $s(t) = \dfrac{-1}{8}(1-e^{-8t})u(t)$

(D) $s(t) = \dfrac{1}{8}(1-e^{8t})u(t)$

Q.2 Define the area under a continuous time signal V(t) is $A_v = \int_{-\infty}^{\infty} V(t)dt$ if y(t)=x(t)*h(t) ,

then A_y=?

(A) $A_x + A_h$ (B) $A_x A_h$ (C) $A_x - A_h$ (D) $(A_x + A_h)^2$

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Q.3 Consider the following two signals

$x(n) = (-1)^n \; ; 0 \leq n \leq 4$
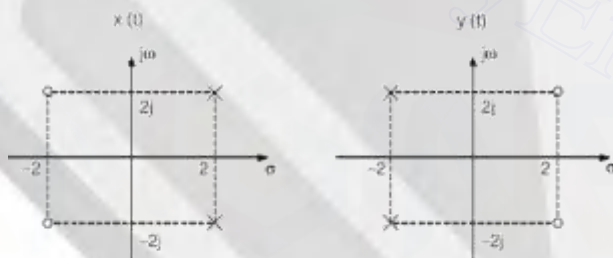
$h(n) = (2)^n \; ; 0 \leq n \leq 3$

the convolution is defined as $y(n) = x(n)*h(n)$ then $y(2)$ & $y(3)$ respectively ....

(A) 5, 3   (B) 3, 5   (C) -3, 5   (D) 3, -5

Q.4 Let $x(n) = \{2, 5, 0, 4\}$; $h(n)=\{4, 1, 3\}$ where $x(n)$ is input signal of a discrete system and $h(n)$ is the impulse response of the same then. The output $y(n)$ of the system is

(A) $y(n) = \{8, 11, 12, 22, 4, 12\}$

(B) $y(n) = \{8, 11, 22, 4, 12, 12\}$

(C) $y(n) = \{8, 22, 11, 31, 4, 12\}$

(D) $y(n) = \{8, 22, 11, 4, 31, 12\}$

**Q.5** Signals $x(t)$ and $y(t)$ have the following pole-zero diagrams

The signal $g(t)$ and $h(t)$ defined as $g(t)=x(t)e^{-3t}$ and $h(t)=y(t)* e^{-t}u(t)$. If $g(t)$ and $h(t)$ are both absolutely integrable, then

(A) $g(t)$ is left sided and $h(t)$ is right sided

(B) $g(t)$ is right sided and $h(t)$ is left sided

(C) both $g(t)$ and $h(t)$ are right sided

(D) both $g(t)$ and $h(t)$ are left sided

Q.6 The impulse response of a causal LTI system is given as $h(t)=u(t)-u(t-6)$. The input to this system shown below

The output of the system at t=2 sec is

(A)1   (B)2

(C)3   (D)4

17
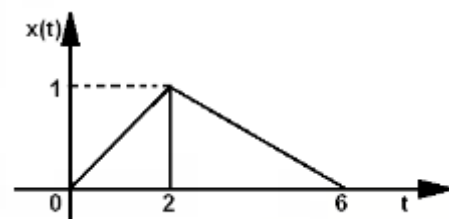
Q.7 The input x(n) of a discrete system is given by $x(n) = \{3, -4, 6, -2\}$ and impulse response $h(n) = \{6, -2, 6\}$. The number of samples in y(n) and y(0)are
(A) 4, 10   (B) 4, 18   (C)6, 18   (D) 6, -30
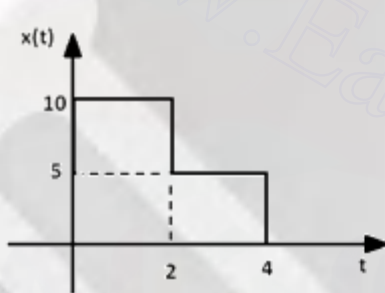
Type 8: Convolution by Graph

For Concept, refer to Signals and Systems K-Notes, LTI Systems

Common Tip:

Flip the function by which the convolution can be easier to save time like unit step function.

Sample Problem 8:

Find the response, when the function x(t) convolve with h(t)

(A) $y(t) = \begin{cases} 1 & t<0 \\ 10t^2 & 0 \le t < 2 \\ 10+5t & 2 \le t < 4 \\ 40 & 4 \le t \end{cases}$

(B) $y(t) = \begin{cases} 0 & t<0 \\ 10t & 0 \le t < 2 \\ 10+5t & 2 \le t < 4 \\ 30 & 4 \le t \end{cases}$

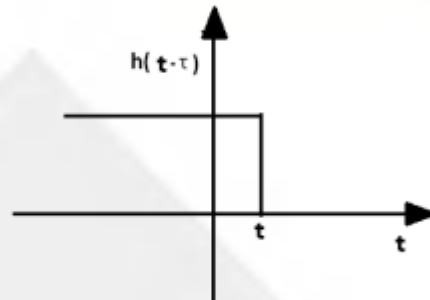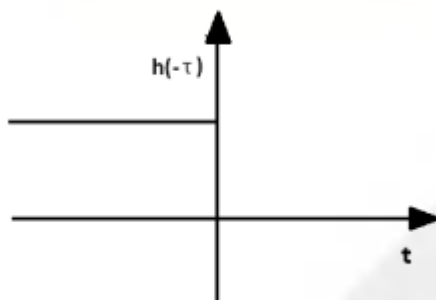(C) $y(t) = \begin{cases} 0 & t<0 \\ 10t & 0 \le t < 2 \\ 10t+5t^2 & 2 \le t < 4 \\ 30 & 4 \le t \end{cases}$

(D)None

Solution: (B) is correct option
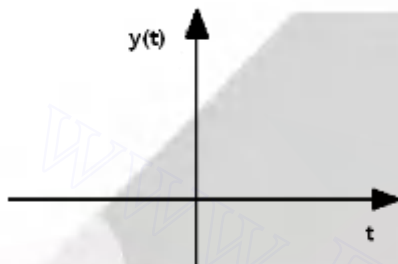
Convolution of x(t) and h(t)

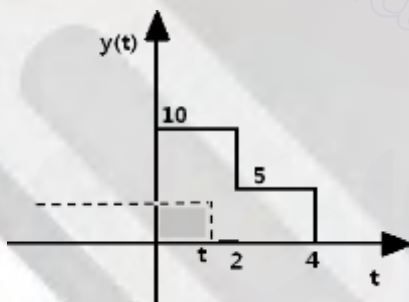$$Y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

for t<0

y(t)=0

for $0 \leq t < 2$

$$y(t) = \int_0^t 10 \times 1 d\tau = 10t$$

for $2 \leq t < 4$
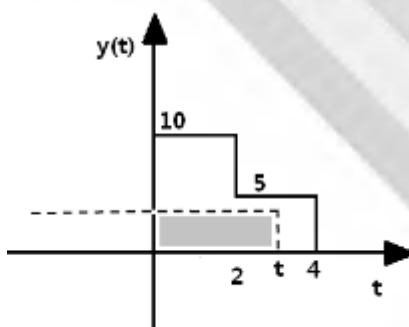
$$y(t) = \int_0^2 10 \times 1 d\tau + \int_2^t 5 \times 1 d\tau = 10 + 5t$$

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for $4 \leq t$
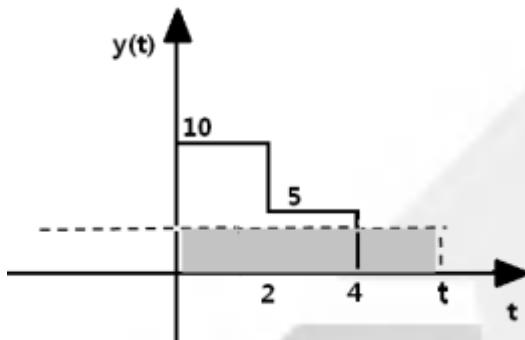
$$y(t) = \int_0^2 10 \times 1 d\tau + \int_2^4 5 \times 1 d\tau = 30$$

Unsolved Problems:

Q.1 Two signals x(t) and y(t) are expressed as follows
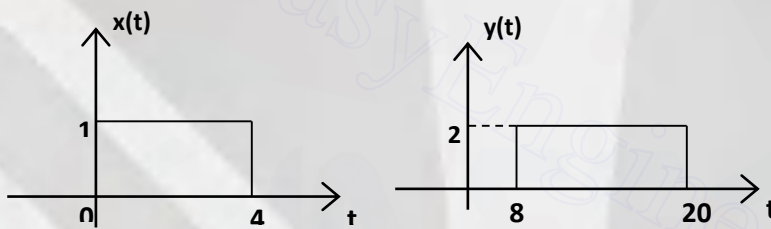
y(t) can be expressed in terms of x(t) as

(A) $y(t) = -2x\left(\dfrac{t}{3} - \dfrac{8}{3}\right)$

(B) $y(t) = 2x\left(\dfrac{t}{3} - \dfrac{8}{3}\right)$

(C) $y(t) = -2x\left(\dfrac{t}{3} - 8\right)$

(D) $y(t) = 2x\left(\dfrac{t}{3} - 8\right)$

Q.2 The convolution of the signals x(t) & h(t) shown in fig. at t = 3 is

(A) 0   (B) 1   (C) 2   (D) 4

Q.3 Given input of system $x(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$ and impulse response h(t) is shown in figure below . Then the output of the system is ?

(A)

(B)

(C)
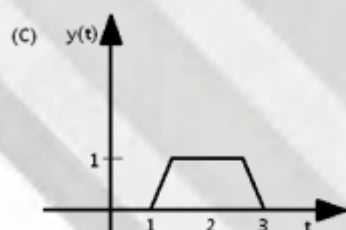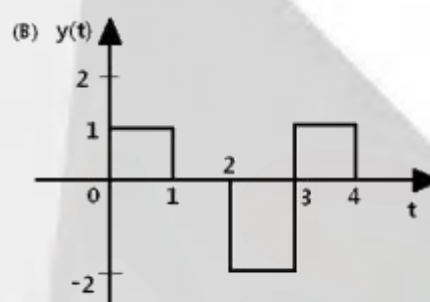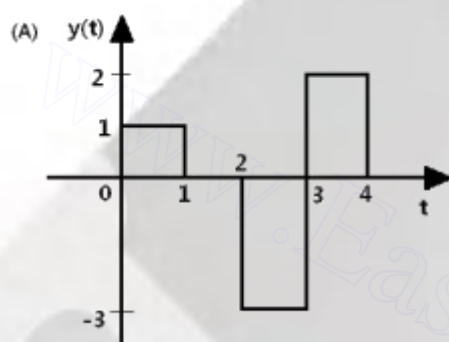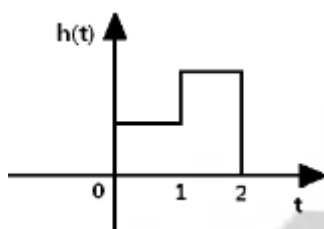
(D) None

Q.4 The graph shown below represents a wave form obtained by convolving two rectangular waveform of duration
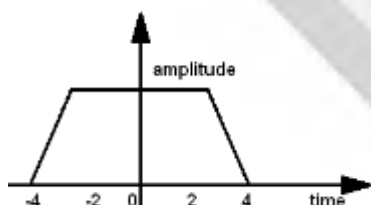
(A) 4 units each

(B) 4 and 2 units respectively

(C) 6 and 3 units respectively

(D)6 and 2 units respectively

Q.5 The input x(t) to a linear time invariant system and the impulse response h(t) of the system is shown below

The output of the system is zero everywhere except for the

(A)1<t<6   (B)0<t<4   (C)0<t<5   (D)1<t<5

Type 9: Continuous Time Fourier Series

For Concept, refer to Signals and Systems K-Notes, Continuous Time Fourier Series

Common Mistake:

Remember to divide by time period while calculating Fourier Series Coefficients.

Sample Problem 9:

The T.F.S. of the signal x(t) is
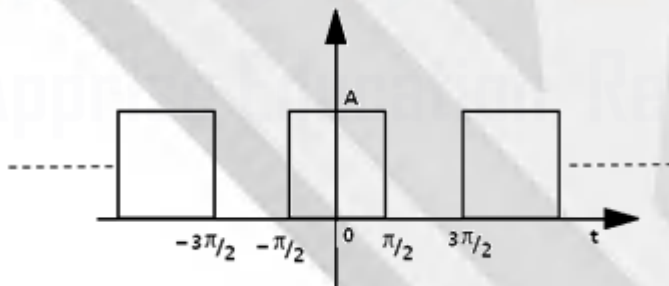
(A) $\dfrac{A}{2} + \dfrac{2A}{\pi}\left(\sin t - \dfrac{1}{3}\sin 3t + \dfrac{1}{5}\sin 5t + \ldots\ldots\ldots \right)$

(B) $\dfrac{2A}{\pi}\left(\sin t - \dfrac{1}{3}\sin 3t + \dfrac{1}{5}\sin 5t + \ldots\ldots\ldots\ldots \right)$

(C) $\dfrac{A}{2} + \dfrac{2A}{\pi}\left(\cos t - \dfrac{1}{3}\cos 3t + \dfrac{1}{5}\cos 5t + \ldots\ldots\ldots \right)$

(D) $\dfrac{A}{2} + \dfrac{2A}{\pi}\left(\sin t + \cos t + \dfrac{1}{3}\sin 3t + \dfrac{1}{3}\sin 3t + \ldots. \right)$

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Solution: (C) is correct option

For the given signal, Fourier Series $x(t) = a_0 + \sum\limits_{n=1}^{\infty} \left\{ a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right\}$.

Period of the signal $T_0 = 2\pi$, $\omega_0 = 1$

$$a_0 = \frac{1}{T} \int\limits_{-\frac{T}{2}}^{\frac{T}{2}} x(t) dt = \frac{1}{2\pi} \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A\, dt = \frac{A}{2}$$

$$a_n = \frac{2}{T} \int\limits_0^{T_0} x(t) \cos(n\omega_0 t) dt = \frac{2}{2\pi} \left\{ \int\limits_0^{\frac{\pi}{2}} A\cos(n\omega_0 t) dt + \int\limits_{\frac{3\pi}{2}}^{2\pi} A\cos(n\omega_0 t) dt \right\} = \frac{A}{\pi} \left\{ \frac{\sin(\frac{n\pi}{2})}{n} - \frac{\sin\left(\frac{3n\pi}{2}\right)}{n} \right\} = \frac{A}{n\pi} \left\{ 2\sin\left(\frac{n\pi}{2}\right) \right\}$$

$$b_n = \frac{2}{T} \int\limits_0^{T_0} x(t) \sin(n\omega_0 t) dt = \frac{2}{2\pi} \left\{ \int\limits_0^{\frac{\pi}{2}} A\sin(n\omega_0 t) dt + \int\limits_{\frac{3\pi}{2}}^{2\pi} A\sin(n\omega_0 t) dt \right\} = \frac{A}{\pi} \left\{ \frac{1 - \cos(2\pi n)}{n} \right\} = 0$$

Hence $x(t) = \dfrac{A}{2} + \dfrac{2A}{\pi} \left(\cos t - \dfrac{1}{3}\cos 3t + \dfrac{1}{5}\cos 5t + \ldots\ldots\ldots \right)$

Unsolved Problems:

Q.1 Fourier series coefficient of the wave form is
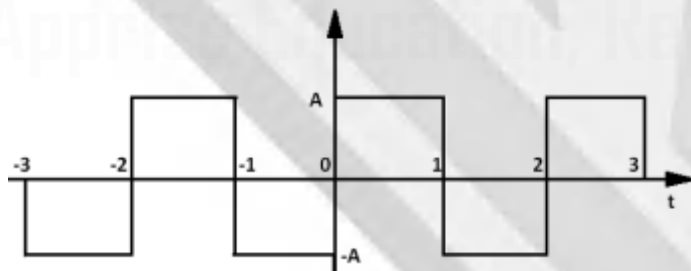
(A) $\dfrac{A}{k\pi}\left[1 - (-1)^k \right]$
(B) $\dfrac{A}{k\pi}\left[1 + (-1)^k \right]$
(C) $\dfrac{A}{jk\pi}\left[1 - (-1)^k \right]$
(D) $\dfrac{A}{jk\pi}\left[1 + (-1)^k \right]$

Q.2 f(x), shown in the figure is represented by $f(x) = a_0 + \sum_{n=1}^{\infty} \{a_n \cos(nx) + b_n \sin(nx)\}$. The value of a_0 is
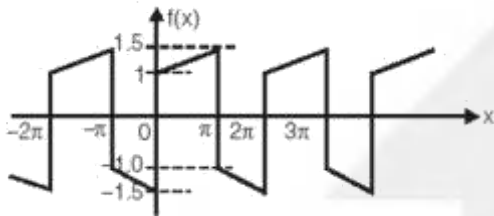

(A) 0   (B) $\pi/2$ (C) π (D) 2π

Q.3 Let $x_1(t)$ and $x_2(t)$ be continuous time periodic signal with fundamental frequency ω_1 and ω_2, Fourier series coefficient C_n and d_n respectively.

Given that $x_2(t) = x_1(t-1) + x_1(1-t)$

The Fourier coefficient d_n will be

(A) $\left(C_n - jC_{-n}\right)e^{-j\omega_1 n}$ (B) $\left(C_{-n} - C_n\right)e^{j\omega_1 n}$

(C) $\left(C_n + jC_{-n}\right)e^{-j\omega_1 n}$ (D) $\left(C_{-n} + C_n\right)e^{-j\omega_1 n}$

Q.4 Determine the time signal corresponding to the magnitude and phase spectra as shown in the figure with $\omega_0 = \pi$
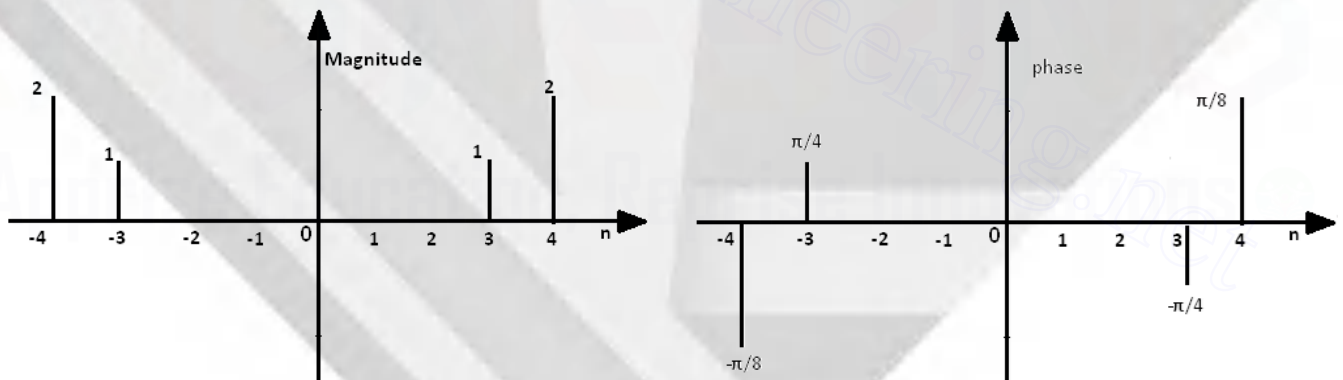
(A) $x(t) = 4\sin\left(4\pi t + \dfrac{\pi}{8}\right) + 2\cos\left(3\pi t - \dfrac{\pi}{4}\right)$

(B) $x(t) = 4\sin\left(2\pi t + \dfrac{\pi}{8}\right) + 2\cos\left(3\pi t - \dfrac{\pi}{4}\right)$

(C) $x(t) = 4\cos\left(4\pi t + \dfrac{\pi}{8}\right) + 2\cos\left(3\pi t - \dfrac{\pi}{4}\right)$

(D) $x(t) = 4\cos\left(2\pi t + \dfrac{\pi}{8}\right) + 2\sin\left(3\pi t - \dfrac{\pi}{4}\right)$

Q.5 The exponential Fourier series of a certain periodic function is given as

$$f(t) = (2+2j)e^{-j3t} + 2je^{-jt} + 3 - 2je^{jt} + (2-2j)e^{j3t}$$

Find the Fourier series of this function

(A) $f(t) = 3 + 4\cos\left(t - \dfrac{\pi}{2}\right) + 2\sqrt{2}\cos\left(3t + \dfrac{\pi}{4}\right)$

(B) $f(t) = 3 + 4\cos\left(t - \dfrac{\pi}{2}\right) + 4\sqrt{2}\cos\left(3t - \dfrac{\pi}{4}\right)$

(C) $f(t) = 3 + 4\sqrt{2}\cos\left(t - \dfrac{\pi}{2}\right) + 4\cos\left(3t - \dfrac{\pi}{4}\right)$

(D) $f(t) = 3 + 4\cos\left(t + \dfrac{\pi}{2}\right) + 4\sqrt{2}\cos\left(3t + \dfrac{\pi}{4}\right)$

Type 10: Continuous Time Fourier Transform

For Concept refer to Signals and Systems K-Notes, Fourier Transform

Sample Problem 10:

Find the Fourier transform of the signal $x(t) = te^{-at}u(t)$: a>0

(A) $\dfrac{1}{a+j\omega}$
(B) $\dfrac{a}{a+j\omega}$
(C) $\dfrac{\omega}{(a+j\omega)^2}$
(D) $\dfrac{1}{(a+j\omega)^2}$

Solution: (D) is correct option

$$X(\omega) = \int_{-\infty}^{\infty} te^{-at}u(t)e^{-j\omega t}dt = \int_{0}^{\infty} te^{-at}e^{-j\omega t}dt = \int_{-\infty}^{\infty} te^{-(a+j\omega)t}dt$$

$$X(\omega) = t\left(\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right)\Bigg|_{0}^{\infty} - \left(\frac{e^{-(a+j\omega)t}}{(a+j\omega)^2}\right)\Bigg|_{0}^{\infty}$$

$$X(\omega) = 0 - \left(-\frac{1}{(a+j\omega)^2}\right) = \frac{1}{(a+j\omega)^2}$$

Unsolved Problems:

Q.1 The Fourier transform $X(\omega)$ of the signal $x(t) = \text{sgn}(t)$ is
(A) $X(\omega) = 2/j\omega$
(B) $X(\omega) = -2/j\omega$
(C) $X(\omega) = j\omega - 2/j\omega$
(D) $X(\omega) = \pi\delta(\omega) + 1/j\omega$

Q.2 Let the signal x(t) have the fourier transform X(ω). Consider the signal $y(t) = \dfrac{d\left[x(t-t_d)\right]}{dt}$, where t_d is arbitrary delay. The magnitude of the fourier transform of y(t) is?

(A) $\left|\omega\right|\left|X(\omega)\right|$　　(B) $\omega\left|X(\omega)\right|$　　(C) $\omega^2\left|X(\omega)\right|$　　(D) $\left|X(\omega)\right|e^{-j\omega t_d}$

Q.3 If a signal x(t)=1+cos(2πft)+cos(6 πft) is fourier transformed , the number of spectral lines in the Fourier transform will be ?

(A)3　　　　(B)4　　　　(C)5　　　　(D)6

Q.4 if $x(t)\xleftarrow{\text{FT}}X(\omega)$ the F.T. of y(t)=x(t-1)e^{jt} is

(A) $X(\omega-1)e^{-j(\omega-1)}$　(B) $X(\omega+1)e^{-j(\omega+1)}$　(C) $X(\omega)e^{-j(\omega-1)}$　(D) $X(\omega+1)e^{-j\omega}$

Q.5 Fourier transform of $f(\dfrac{1}{a}+b)$ is

(A) $\dfrac{1}{|a|}e^{jab\omega}F(a\omega)$　(B) $|a|e^{jab\omega}F(a\omega)$　(C) $|a|e^{-jab\omega}F(b\omega)$　(D) $\dfrac{1}{|a|}e^{-jab\omega}F(b\omega)$

Type 11: Fourier Transform by Graph

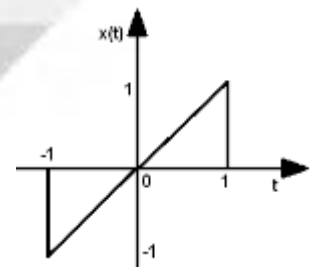*For Concept refer to Signals and Systems K-Notes, Fourier Transform.*

Common Mistake:
Remember to consider the impulse function while taking the derivative of the rectangular function at the edges.

Sample Problem 11:
 The Fourier transform of the given signal x(t) is

(A) $4\pi j\left(\dfrac{\cos\omega}{\omega}-\dfrac{\sin\omega}{\omega}\right)$

(B) $2j\left(\dfrac{\cos\omega}{\omega}-\dfrac{\sin\omega}{\omega^2}\right)$

(C) $4\pi j\left(\dfrac{\cos\omega}{\omega^2}-\dfrac{\sin\omega}{\omega}\right)$

(D) $2j\left(\dfrac{\cos\omega}{\omega^2}-\dfrac{\sin\omega}{\omega}\right)$

Solution: (B) is correct option

Method 1

equation of $x(t) = t[u(t+1) - u(t-1)] = (t+1)u(t+1) - u(t+1) - [(t-1)u(t-1) + u(t-1)]$

Taking Fourier Transfer

$$X(j\omega) = -\frac{e^{j\omega}}{\omega^2} - \frac{e^{j\omega}}{j\omega} + \frac{e^{-j\omega}}{\omega^2} - \frac{e^{-j\omega}}{j\omega} = \frac{1}{\omega^2}[-2j\sin\omega] - \frac{2}{j\omega}\cos\omega$$

$$X(j\omega) = 2j\left(\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega^2}\right)$$
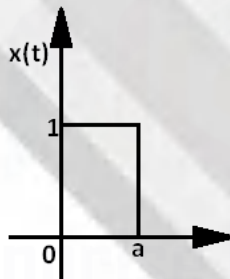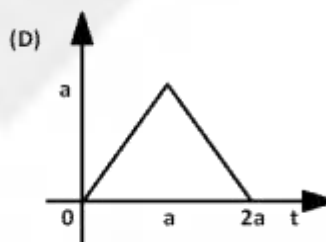
Method 2

equation of $x(t) = t[u(t+1) - u(t-1)]$

Taking Fourier Transform

$$X(j\omega) = j\frac{dF.T.[u(t+1) - u(t-1)]}{d\omega} = j\frac{d}{d\omega}(\frac{2\sin\omega}{\omega}) = j(\frac{2\omega\cos\omega - 2\sin\omega}{\omega^2})$$

$$X(j\omega) = 2j(\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega^2})$$

Unsolved Problems:
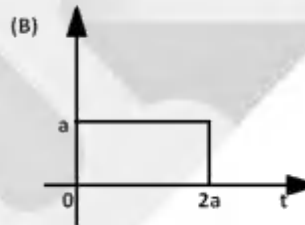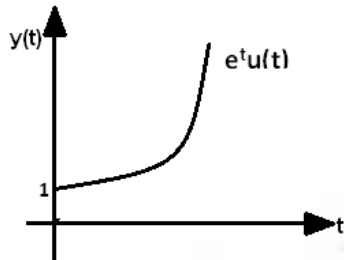
Q.1 The signal x(t) is shown as

Then the inverse Laplace transform of $X^2(s)$ is?

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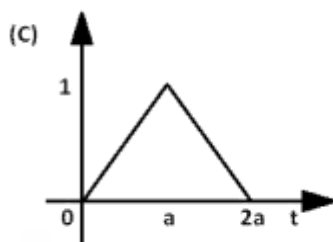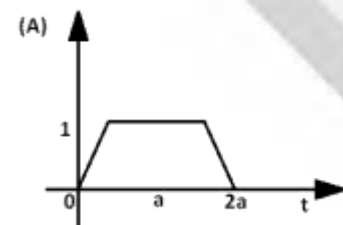
Q.2 For the signal shown below

(A) Only Fourier transform exists

(B) Only Laplace transform exists

(C) Both Laplace and Fourier transforms exists

(D) Neither Laplace nor Fourier transforms exists

Q.3 Find the Inverse Fourier transform of X(ω) for the magnitude and phase spectra of X(ω)
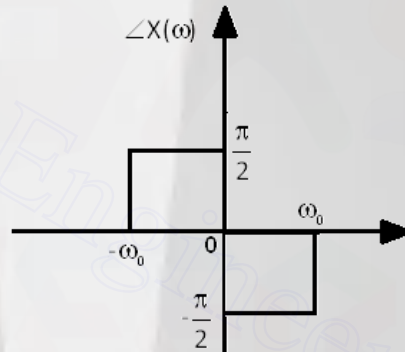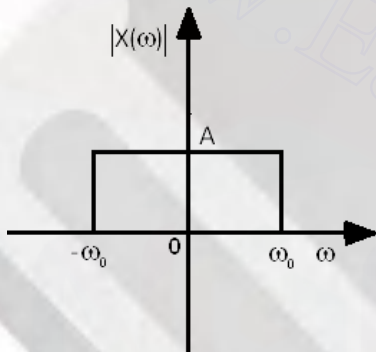
(A) $\dfrac{A}{\pi}\left[1-\cos\omega_0 t\right]$

(B) $\dfrac{A}{\pi t}\left[1-\cos\omega_0 t\right]$

(C) $\dfrac{A}{\pi}\left[1-\sin\omega_0 t\right]$

(D) $\dfrac{A}{\pi t}\left[1-\sin\omega_0 t\right]$

Q.4 Find the inverse Fourier transform of the spectra F(ω) depicted below

(A) $\dfrac{4}{\pi}\text{sinc}^2(t)\cos 4t$

(B) $\dfrac{2}{\pi}\text{sinc}^2(t)\cos 4t$

(C) $\dfrac{2}{\pi}\text{sinc}^2(t)\sin 4t$

(D) $\dfrac{4}{\pi}\text{sinc}^2(t)\sin 4t$
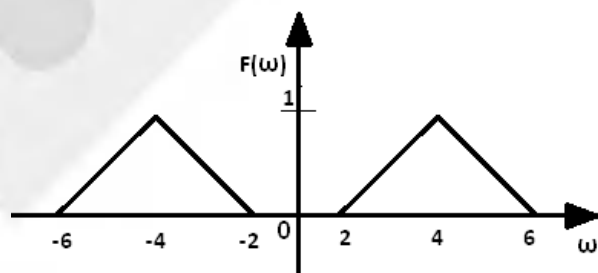
Q.5 Determine the Fourier transform for the waveform shown below
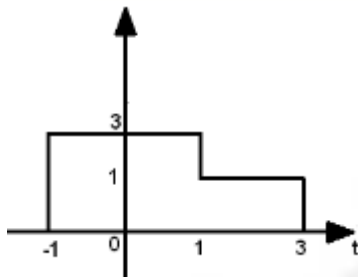
(A) $\dfrac{\sin(2\pi f)}{\pi f}(3+e^{-2j\omega})$

(B) $\dfrac{\sin(2\pi f)}{\pi f}(3-e^{-2j\omega})$

(C) $\dfrac{\sin(2\pi f)}{\pi f}(3+e^{2j\omega})$

Type 12: Properties of Fourier Transform

For Concept refer to Signals and Systems K-Notes, Fourier Transform.

Common Mistake:

Remember the duality property correctly for functions like rectangular and triangular.

Sample Problem 12:

Let $x(t)=\text{rect}\left(t-\dfrac{1}{2}\right)$ where rect(x)=1 for $-\dfrac{1}{2}<x<\dfrac{1}{2}$ and zero other wise .

Then if $\text{sinc}(x)=\dfrac{\sin(\pi x)}{\pi x}$, The fourier transform of x(t)+x(-t) will be given by

(A) $\text{sinc}\left(\dfrac{\omega}{2\pi}\right)$

(B) $2\,\text{sinc}\left(\dfrac{\omega}{2\pi}\right)$

(C) $2\,\text{sinc}\left(\dfrac{\omega}{2\pi}\right)\cos(\dfrac{\omega}{2})$

(D) $\text{sinc}\left(\dfrac{\omega}{2\pi}\right)\cos(\dfrac{\omega}{2})$

Solution: (B) is correct option

For $x(t)=\text{rect}\left(t-\dfrac{1}{2}\right)$
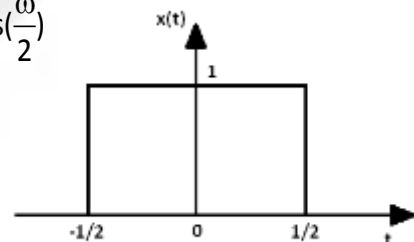
$$X(f), X(\omega) = 1 \operatorname{sinc}(f) = \operatorname{sinc}\left(\frac{\omega}{2\pi}\right) = \frac{\sin(\pi f)}{\pi f} = \frac{\sin(\frac{\omega}{2})}{\frac{\omega}{2}} = sa(\frac{\omega}{2})$$

Note that Sinc and Sa functions are even,

i.e. Sinc(λ)=Sinc(-λ)

   Sa(λ)=Sa(-λ)

Using Time Reversal property of F.T. :

$$x(-t) \xrightarrow{FT} X(-f) \text{ or } X(-\omega)$$

$$\operatorname{sinc}\left(-\frac{\omega}{2\pi}\right) = \operatorname{sinc}\left(\frac{\omega}{2\pi}\right) = X(\omega)$$

$$\therefore x(t) + x(-t) \xrightarrow{FT} X(\omega) + X(-\omega)$$

$$= 2X(\omega) = 2\operatorname{Sinc}\left(\frac{\omega}{2\pi}\right)$$

Unsolved Problems:

Q.1 The Fourier Transform of. $x(t) = \dfrac{4\cos 2t}{t^2 + 1}$

(A) $\dfrac{3\omega}{\omega^2 + 1}$

(B) $\dfrac{4\omega}{4\omega^2 + 1}$

(C) $\dfrac{2\omega}{\omega^2 + 4}$

(D) None of the above

Q.2 The Fourier Transform of. $x(t) = \dfrac{2}{t^2 + 1}$

(A) $\dfrac{\sin \omega}{2}$

(B) $\dfrac{\cos \omega}{2}$

(C) $2\pi e^{-4|\omega|}$

(D) $2\pi e^{-|\omega|}$

Q.3 A $-\dfrac{\pi}{2}$ phase shifter is defined by the frequency response

$$H(j\omega) = \begin{cases} e^{-j\frac{\pi}{2}} & \omega > 0 \\ e^{j\frac{\pi}{2}} & \omega < 0 \end{cases}$$

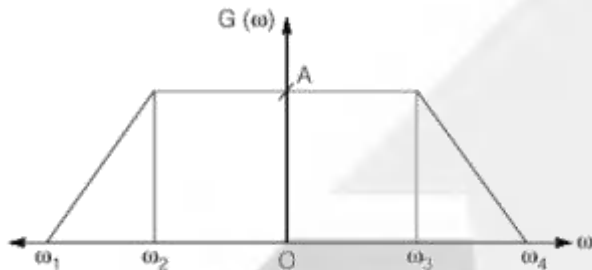Then the impulse response h(t) will be

(A) $\dfrac{1}{\pi t}$

(B) $-\dfrac{1}{\pi t}$

(C) $\dfrac{2}{\pi t}$

(D) $-\dfrac{2}{\pi t}$

Q.4 Consider a signal $g(t) = x_1(t) \times x_2(t)$ where $x_1(t) = \text{sinc}50t$ and $x_2(t) = \text{sinc}100t$

Also $g(t) \xrightarrow{\text{F.T.}} G(\omega)$

The amplitude A of the curve $G(\omega)$ is equal to 10^{-x}. Then x is

(A)1   (B)2   (C)4   (D)5

Q.5 The inverse Laplace transform of $X(s) = \dfrac{4s^2 + 15s + 8}{(s+2)^2(s-1)}$. If the Fourier transform of x(t)

exists is ?

(A) $e^{-2t}u(t) + 2te^{-2t}u(t) - 3e^{-t}u(-t)$ (B) $e^{-2t}u(-t) - 2te^{-2t}u(t) + 3e^{t}u(t)$

(C) $-e^{-2t}u(t) - 2te^{-2t}u(t) + 3e^{-t}u(-t)$ (D) None

Type 13: Initial Value in Fourier Transform

For Concept refer to Signals and Systems K-Notes, Fourier Transform

Sample Problem 13:

For the signal x(t) shown in figure find X(0) and $\int\limits_{-\infty}^{\infty} X(\omega)d\omega$?

(A)7,4π

(B)21,6π

(C)69,π
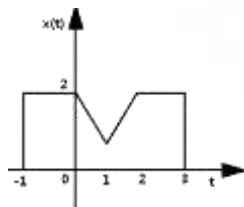
(D)None

Solution: (A) is correct option

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$$

Put t=0 in above equation

$$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) d\omega$$

Hence $\int_{-\infty}^{\infty} X(\omega) d\omega = 2\pi x(0) = 2\pi \times 2 = 4\pi$

and $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$

put ω=0 in above equation

$$X(0) = \int_{-\infty}^{\infty} x(t) dt = \text{area of the signal} = 7$$

Unsolved Problems:

Q.1 For the signal x(t) shown below,

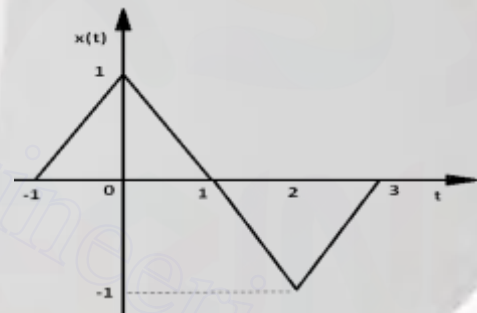Find $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ and $\int_{-\infty}^{\infty} X(\omega)e^{-2j\omega} d\omega$

(A) $\frac{7\pi}{3}, -\pi$ (B) $\frac{8\pi}{3}, -2\pi$

(C) $\pi, -2\pi$ (D) $\frac{8\pi}{3}, -\pi$

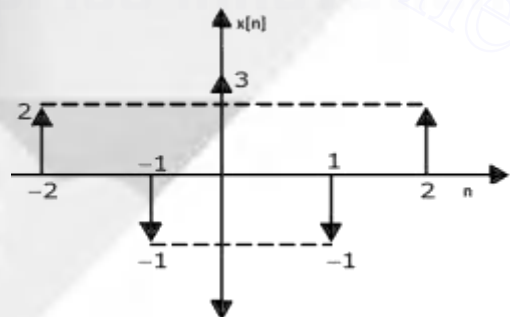**Q.2** The sequence x(n) represents output of system in discrete time domain.
The value of X(π) would be?

(A)3

(B)6

(C)9

(D)12

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Type 14: Laplace Transform

For Concept refer to Signals and Systems K-Notes, Laplace Transform

Common Mistake:
While Calculating Inverse Laplace Transform, take care about Region of Convergence of the Transform like Laplace Transform may be same for both left sided signal and Right Sided Signal.

Sample Problem 14:

The Laplace transform of $(t^2 - 2t)u(t-1)$ is

(A) $\dfrac{2}{s^3}e^{-s} - \dfrac{2}{s^2}e^{-s}$ (B) $\dfrac{2}{s^3}e^{-2s} - \dfrac{2}{s^2}e^{-s}$ (C) $\dfrac{2}{s^3}e^{-s} - \dfrac{1}{s}e^{-s}$ (D) None of the above

Solution: (C) is correct option

Let $x(t) = (t^2 - 2t)u(t-1) = (t-1)^2 u(t-1) - u(t-1)$

Use the pairs and properties of LT:

$u(t) \xrightarrow{\text{LT}} \dfrac{1}{s}, \quad u(t-1) \xrightarrow{\text{LT}} \dfrac{1}{s}e^{-s}$

$tu(t) \xrightarrow{\text{LT}} \dfrac{1}{s^2}, \quad t^2 u(t) \xrightarrow{\text{LT}} \dfrac{2}{s^3}$

$(t-1)^2 u(t-1) \xrightarrow{\text{LT}} \dfrac{2}{s^3}e^{-s}$

$\therefore X(s) = \dfrac{2}{s^3}e^{-s} - \dfrac{1}{s}e^{-s}$

Unsolved Problems:

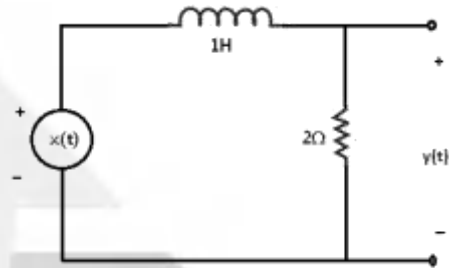**Q.1** The O/p y(t) of a causal LTI system is related to the I/p x(t) as

$$\dfrac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau - x(t)$$

where x(t) = $e^{-t}U(t) + 3\delta(t)$ then impulse response is

(A) $\dfrac{1}{9}e^{-t}U(t) + \dfrac{17}{9}e^{-10t}U(t)$ (B) $\dfrac{17}{9}e^{-t}U(t) + \dfrac{1}{9}e^{-10t}U(t)$

(C) $e^{-t}U(t) + e^{-10t}U(t)$ (D) $\delta(t) - e^{-t}U(t)$

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Q.2 The output of the system shown in figure below .If the input x(t)= te^{-2t}u(t) is ?

(A) $t^2e^{-2t}u(t)$

(B) $te^{-2t}u(t)$

(C) $\dfrac{t^2}{2}e^{-2t}u(t)$

(D) $\dfrac{t}{2}e^{-2t}u(t)$

Q.3 The Laplace transform of $(\sin\omega t)u(t-t_0)$ is

(A) $\dfrac{e^{-st_0}}{s^2+\omega^2}\sin\left\{\omega t_0 + \tan^{-1}(\dfrac{\omega}{s})\right\}$

(B) $\dfrac{e^{-st_0}}{\sqrt{s^2+\omega^2}}\sin\left\{\omega t_0 + \tan^{-1}(\dfrac{\omega}{s})\right\}$

(C) $\dfrac{e^{-st_0}}{\sqrt{s^2+\omega^2}}\sin\left\{\omega t_0 + \tan^{-1}(\dfrac{s}{\omega})\right\}$

(D) $\dfrac{e^{-st_0}}{\sqrt{s^2+\omega^2}}\sin\left\{\omega t_0 - \tan^{-1}(\dfrac{s}{\omega})\right\}$

Q.4 The Laplace transform of a signal x(t) is

$X(s) = \dfrac{9}{(s-1)(s+2)^2}$ ROC : -2<Re(s)<1 . The signal x(t) is?

(A) $e^t u(-t) - e^{-2t}u(-t) - 3te^{-2t}u(-t)$

(B) $e^t u(t) + e^{-2t}u(-t) - 3te^{-2t}u(t)$

(C) $-e^t u(t) - e^{-2t}u(t) + 3te^{-2t}u(t)$

(D) $-e^t u(-t) - e^{-2t}u(t) - 3te^{-2t}u(t)$

Q.5 What is the inverse Laplace transform of $X(s) = \log\left(\dfrac{s^2+a^2}{s^2-b^2}\right)$

(A) $x(t) = \dfrac{2}{t}\left[\cosh bt - \cosh at\right]$

(B) $x(t) = \dfrac{2}{t}\left[\sinh at - \sinh bt\right]$

(C) $x(t) = \dfrac{2}{t}\left[\cosh bt - \cos at\right]$

(D) $x(t) = \dfrac{2}{t}\left[\sinh at - \sin bt\right]$

Type 15: Initial and Final Value Theorem

For Concept, refer to Signals and Systems K-Notes, Laplace Transform.

Common Mistake:

Check for Stability of a signal before applying Final Value Theorem.

Sample Problem 15:

The unit impulse response of a second order under-damped system starting from rest is given by $C(t) = 12.5e^{-6t}\sin 8t, \ t \geq 0$. The steady state value of the unit step response of the system is equal to

(A)0   (B)0.25   (C)0.5   (D)1.0

Solution: (D) is correct option

Impulse response

$h(t) = 12.5e^{-6t}\sin(8t), \ t \geq 0$

$\sin(8t)u(t) \xrightarrow{\text{L.T.}} \dfrac{8}{s^2 + 64}$

$e^{-6t}\sin(8t)u(t) \xrightarrow{\text{L.T.}} \dfrac{8}{(s+6)^2 + 64}$

$h(t) \xrightarrow{\text{L.T.}} H(s) = \dfrac{100}{(s+6)^2 + 64}$

If the step response is y(t)

$y(t) = \displaystyle\int_{-\infty}^{t} h(t)dt,$

$Y(s) = \dfrac{H(s)}{s} = \dfrac{100}{s\left[(s+6)^2 + 64\right]}$ (integrating property of LT is used)

$\therefore y_{ss} = \lim_{t \to \infty} y(t) = \lim_{s \to 0}[sY(s)] = \dfrac{100}{100} = 1$ (Final value theorem of LT is used)

Unsolved Problems:

Q.1 $L\{f(t)\} = \dfrac{2(s+1)}{(s^2 + 2s + 5)}$ then f(0) and f(∞) are ?

(A)0,2   (B)2,0   (C)0,1   (D)0.4,0

Q.2 Initial and final value of $X(s) = \dfrac{(s+3)}{(s+2)}$ respectively are

(A)0,0   (B)∞,0   (C)1,1.5   (D)1,0

Q.3 Determine the initial and final values of the f(t), Given that $F(s) = \dfrac{(s^2 - 2s + 1)}{(s-2)(s^2 + 2s + 4)}$

(A)0,8   (B)0,0   (C)1,doesn't exist   (D)None

Q.4 The unit step response of a system with the transfer function $X(s) = \dfrac{b(s+a)}{(s+b)}$ is c(t).If

c(0)=2 and c(∞)=10 , then the ratio $\dfrac{a}{b}$ is

(A)4   (B)5   (C)16   (D)25

Q.5 The z-Transform of signal is $X(z) = \dfrac{1}{4}z^{-1}\dfrac{(1-z^{-4})}{(1-z^{-1})^2}$ .What will be tha value of x[∞]?

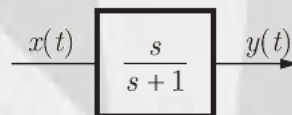(A)1/4   (B)0   (C)1   (D) ∞

Type 16: Magnitude and Phase Response

For Concept refer to Signals and Systems K-Notes, Laplace Transform.

Sample Problem 16:
In the system shown in figure, the input x(t) = sint . In the steady-state, the response y(t) will be

(A) $\dfrac{1}{\sqrt{2}}\sin(t - 45^0)$ (B) $\dfrac{1}{\sqrt{2}}\sin(t + 45^0)$ (C) $\sin(t - 45^0)$ (D) $\sin(t + 45^0)$

Solution: (A) is correct option

System response $H(s) = \dfrac{s}{s+1}$ Amplitude response

$H(j\omega) = \dfrac{j\omega}{j\omega + 1}$ $\left|H(j\omega)\right| = \dfrac{\omega}{\sqrt{\omega^2 + 1}}$

Given input frequency ω=1 rad/sec

SO, $\left|H(j\omega)\right|_{\omega=1} = \dfrac{1}{\sqrt{1^2 + 1}} = \dfrac{1}{\sqrt{2}}$

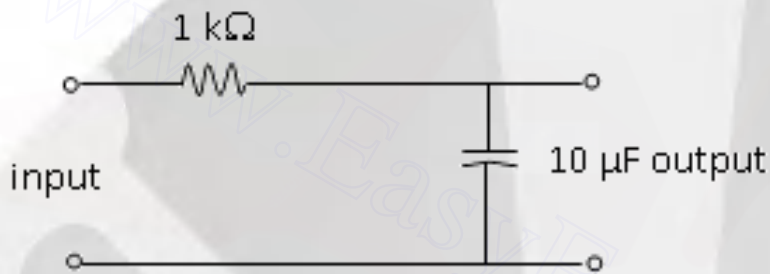Phase response

$$\theta_h(j\omega) = 90^0 - \tan^{-1}(\omega)$$

$$\theta_h(j\omega)\Big|_{\omega=1} = 90^0 - \tan^{-1}(1) = 45^0$$

So the output of the system is

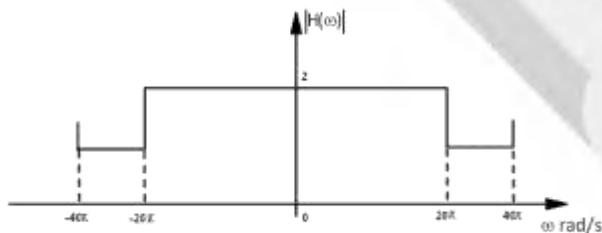$$y(t) = |H(j\omega)| x(t-\theta_h) = \frac{1}{\sqrt{2}} \sin(t - 45^0)$$

Unsolved Problems:

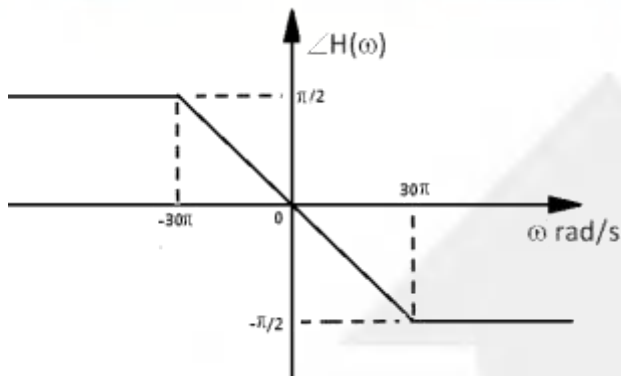**Q.1** In fig. the steady state output corresponding to the input 3 + 4 sin 100t is

(A) $3 + \dfrac{4}{\sqrt{2}} \sin(100t - 45^0)$

(B) $3 + 4\sqrt{2} \sin(100t + 45^0)$

(C) $\dfrac{3}{\sqrt{2}} + \dfrac{4}{\sqrt{2}} \sin(100t + 45^0)$

(D) $\dfrac{3}{\sqrt{2}} + \dfrac{4}{\sqrt{2}} \sin(100t - 45^0)$

Q.2 Consider a distortion less system H(ω) with magnitude and phase response as shown below . If an input signal $x(t) = 2\cos10\pi t + \sin26\pi t$ is given to this system the output will be

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(A) $4\cos 10\pi t + \sin 26\pi t$

(B) $8\cos 10\pi t + \sin 26\pi t$

(C) $4\cos\left(10\pi t - \dfrac{\pi}{6}\right) + \sin\left(26\pi t - \dfrac{13\pi}{30}\right)$

(D) $8\cos\left(10\pi t - \dfrac{\pi}{2}\right) + \sin\left(26\pi t - \dfrac{\pi}{2}\right)$

Q.3 Let a signal $a_1\sin(\omega_1 t + \phi)$ be applied to a stable linear time variant system. Let the corresponding steady state output be represented as $a_2 F(\omega_2 t + \phi_2)$. Then which of the following statement is true?
(A) F is not necessarily a "Sine" or "Cosine" function but must be periodic with $\omega_1 = \omega_2$
(B) F must be a "Sine" or "Cosine" function with $a_1 = a_2$
(C) F must be a "Sine" function with $\omega_1 = \omega_2$ and $\phi_1 = \phi_2$
(D) F must be a "Sine" or "Cosine" function with $\omega_1 = \omega_2$

Q.4 A causal LTI filter has the frequency response $H(j\omega)$ shown below . For the input signal $x(t) = e^{-jt}$, output will be?
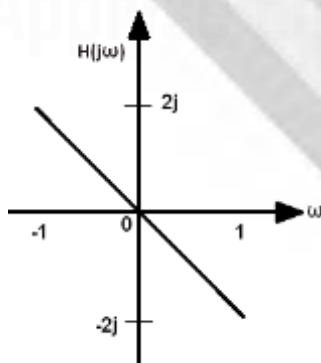
(A) $-2je^{-jt}$ (B) $2je^{-jt}$ (C) $4\pi je^{-jt}$ (D) $-4\pi je^{-jt}$

Q.5 Consider a LTI system with system function $H(s) = \dfrac{s-2}{s^2 + 4s + 4}$. The steady state response of the system is given by (when the excitation is $8\cos 2t$)
(A) $4\cos(2t - 45^0)$ (B) $\sqrt{8}\cos(2t + 45^0)$
(C) $8\cos(2t + 135^0)$ (D) $\sqrt{8}\cos(2t - 135^0)$

Type 17: Z-Transform

For Concept refer to Signals and Systems K-Notes, Z-Transform.

Common Mistake:

Take care of ROC while calculating inverse of Z-Transform.

Sample Problem 17:

Consider the D.E. $y(n) - \frac{1}{3}y(n-1) = x(n)$ and $x(n) = \left(\frac{1}{2}\right)^n u(n)$. Assuming the condition of initial

rest, the solution for y(n): $n \geq 0$ is

(A) $3\left(\frac{1}{3}\right)^n - 2\left(\frac{1}{2}\right)^n$

(B) $-2\left(\frac{1}{3}\right)^n + 3\left(\frac{1}{2}\right)^n$

(C) $\frac{2}{3}\left(\frac{1}{3}\right)^n + \frac{1}{3}\left(\frac{1}{2}\right)^n$

(D) $\frac{1}{3}\left(\frac{1}{3}\right)^n + \frac{2}{3}\left(\frac{1}{2}\right)^n$

Solution: (B) is correct option

Given D.E. $y(n) - \frac{1}{3}y(n-1) = x(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u(n)z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} = \left\{ \frac{1}{1-\frac{1}{2z}} \right\}$$

Taking 'z' Transform and Using time shifting property of 'z' Transform

$$Y(z) - \frac{1}{3}y(z)z^{-1} = X(z) \Rightarrow Y(z)\left\{1 - \frac{1}{3}z^{-1}\right\} = \left\{ \frac{1}{1-\frac{1}{2z}} \right\}$$

$$Y(z) = \left\{ \frac{1}{1-\frac{1}{2z}} \right\}\left\{ \frac{1}{1-\frac{1}{3z}} \right\} = \left\{ \frac{3}{1-\frac{1}{2z}} \right\} - \left\{ \frac{2}{1-\frac{1}{3z}} \right\}$$

Taking inverse 'z' Transform

$$Y(z) = 3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n$$

Unsolved Problems:

Q.1 The 'z' transform X(z) of the signal $x[n] = \left(\dfrac{-1}{4}\right)^n u(-n-3)$ is

(A) X(z) = $\dfrac{-64z^3}{1+4z}$; $|z| > \dfrac{1}{4}$

(B) X(z) = $\dfrac{64z^3}{1-4z}$; $|z| > \dfrac{1}{4}$

(C) X(z) = $\dfrac{-64z^2}{1+4z^2}$; $|z| < \dfrac{1}{4}$

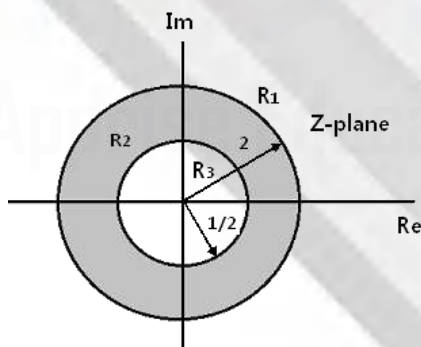(D) X(z) = $\dfrac{-64z^3}{1+4z^2}$; $|z| < \dfrac{1}{4}$

Q.2 Consider three different signal

$$x_1[n] = \left[2^n - \left(\dfrac{1}{2}\right)^n\right]u[n]$$

$$x_2[n] = -2^n u[-n-1] + \left(\dfrac{1}{2}\right)^n u[-n-1]$$

$$x_3[n] = -2^n u[-n-1] - \left(\dfrac{1}{2}\right)^n u[n]$$

Following figure shows the three different region. Choose the correct for the ROC of signal

	R_1	R_2	R_3
(A)	$x_1[n]$	$x_2[n]$	$x_3[n]$
(B)	$x_2[n]$	$x_3[n]$	$x_1[n]$
(C)	$x_1[n]$	$x_3[n]$	$x_2[n]$
(D)	$x_3[n]$	$x_2[n]$	$x_1[n]$

Q.3 H(z) is transfer function of discrete system and has two poles at $z = \dfrac{1}{2} \,\&\, 2$.

H(z) is rational. ROC includes z = 3 / 4,   h(0) = 1 & h(−1) = 4 . Impulse response h(n) of the system is
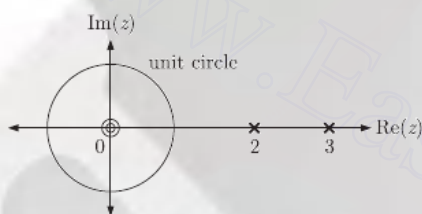
(A) $h(n) = \left(\dfrac{1}{2}\right)^n u(n) + 4^{n+2} u(n-1)$

(B) $h(n) = -\left(\dfrac{1}{2}\right)^n u(n) - 4^{n+2} u(-n-1)$

(C) $h(n) = \left(\dfrac{1}{2}\right)^n u(n) + 4^{n+2} u(-n-1)$

(D) $h(n) = -\left(\dfrac{1}{2}\right)^n u(n) - 4^{n+2} u(-n)$

Q.4 Consider the pole zero diagram of an LTI system shown in the figure which corresponds to transfer function H(z).

Match List I (The impulse response) with List II (ROC which corresponds to above diagram) and choose the correct answer using the codes given below:
{Given that *H*(1) = 1}

List-I (Impulse Response)	List-II (ROC)
P. [(- 4)2n + 6(3)n]u[n]	1. does not exist
Q. (- 4)2nu[n] + (- 6)3nu[- n - 1]	2. z > 3
R. (4)2nu[- n - 1] + (- 6)3nu[- n - 1]	3. z < 2
S. 4(2)nu[- n - 1] + (- 6)3nu[n]	4. 2 < z < 3

Codes :

 P Q R S

(A) 4 1 3 2

(B) 2 1 3 4

(C) 1 4 2 3

(D) 2 4 3 1

Q.5 The input-output relationship of a system is given as y[n] - 0.4y[n - 1] = x[n] where, x[n] and y[n] are the input and output respectively. The zero state response of the system for an input x[n] = (0.4)nu[n] is

(A) $n(0.4)^n u[n]$

(B) $n^2(0.4)^n u[n]$

(C) $(n+1)(0.4)^n u[n]$

(D) $\dfrac{1}{n}(0.4)^n u[n]$

Q.6 The z -transform of $x[n]$ = {2, 4, 5, 7, 0, 1}

(A) $2z^2 + 4z + 5 + 7z + z^3, z \neq \infty$

(B) $2z^{-2} + 4z^{-1} + 5 + 7z + z^3, z \neq \infty$

(C) $2z^{-2} + 4^{z-1} + 5 + 7z + z^3, \ 0 < |z| < \infty$

(D) $2z^2 + 4z + 5 + 7z^{-1} + z^{-3}, \ 0 < |z| < \infty$

Q.7 The system diagram for the transfer function $H(z) = \dfrac{z}{z^2 + z + 1}$, is shown below
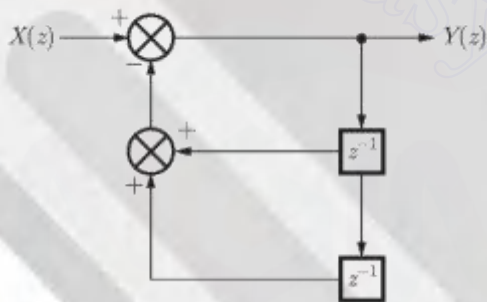
The system diagram is a
(A) Correct solution
(B) Not correct solution
(C) Correct and unique solution
(D) Correct but not unique solution

Type 18: Sampling

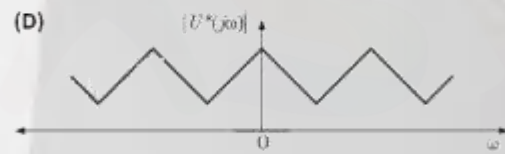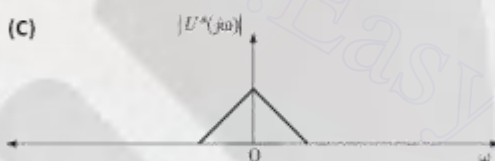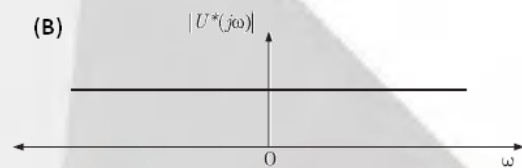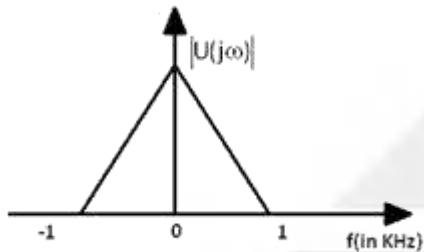*For Concept, refer to Signals and Systems K-Notes, Sampling*

Common Mistake:
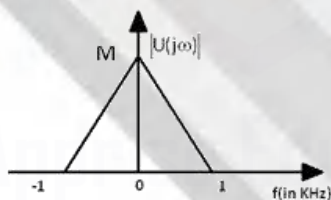*For Band Pass Signals Nyquist Rate is different from Low Pass Signals.*

Sample Problem 18:

The frequency spectrum of a signal is shown in the figure. If this is ideally sampled at intervals of 1 ms, then the frequency spectrum of the sampled signal will be

(A)

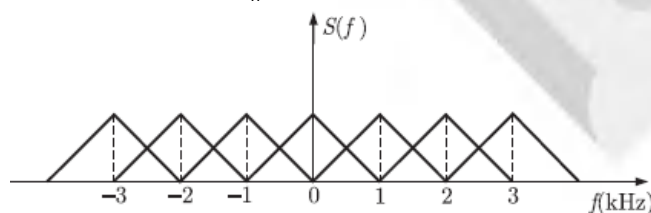(B)

(C)

(D)

Solution: (B) is correct option

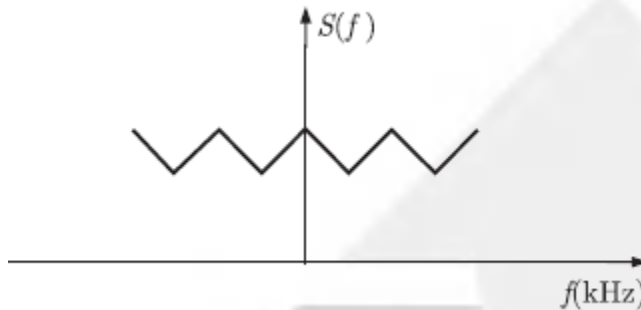Highest frequency of the input signal, f_h=1 KHz as shown in the figure

Sampling interval, T_s=1 ms, f_s=1 KHz => $f_{s=}$ f_h , Therefore Aliasing or overlap of the adjacent spectra occurs in the sampled spectrum because f_s< $2f_h$

The Sampled spectrum :

$$U*(j\omega)=U*(f)=f_s\sum_{n=-\infty}^{\infty}U(f-nf_s)$$

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The resultant spectrum $U*(j\omega)$ is constant for all 'f' as shown in figure below


Unsolved Problems:

Q.1 The Nyquist rate of the signal $x(t) = 3\cos(3000\pi t) + 5\sin(9000\pi t)$
(A) 9 KHz   (B) 4.5 KHz   (C)6 KHz   (D) 2.5 KHz

Q.2 A continuous signal x(t) is obtained at the O/p of an ideal LPF with cut off frequency 1000 π. If impulse train sampling is Performed on x(t), which of the following sampling Periods would guarantee that x(t) can be recovered form its sampled version using an appropriate LPF?

(1) T = 0.5m sec   (2) T = 2m sec   (3) T = 0.1m sec
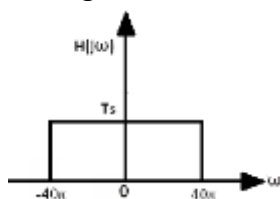
(A) 1, 3   (B) 1, 2, 3   (C) 2, 3   (D) 1, 2

Q.3 Let, m(t)=2cos600πt + 2cos800πt, if it is sampled by rectangular pulse train , as shown in the following figure .
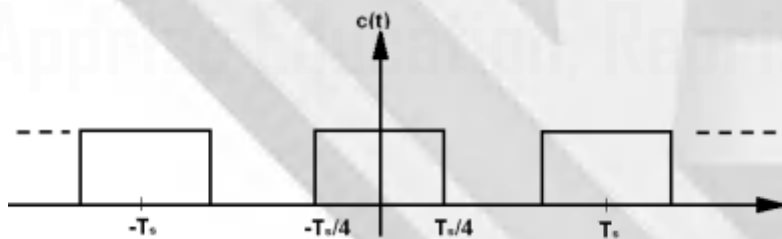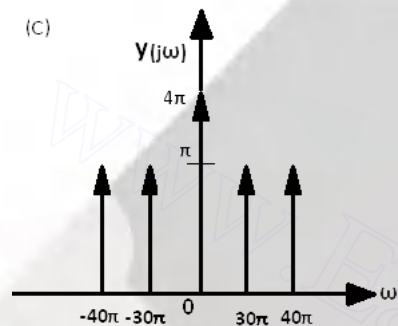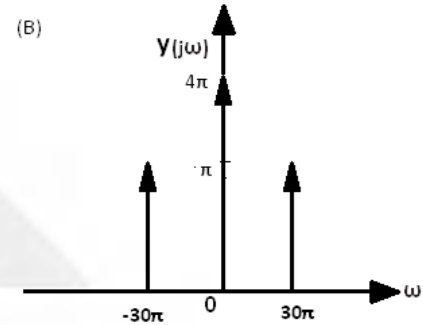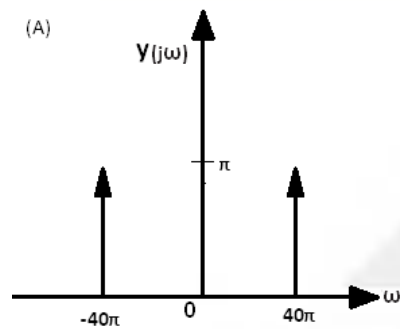

The spectral components in KHz present in sampled signal in frequency range 2KHz to 3KHz
(A)2.3,2.4,2.6,2.7   (B)2.6,2.7   (C)2.3,2.4   (D)2.2,2.4,2.6,2.8

Q.4 A signal $x(t) = 2 + \cos(50\pi t)$ is sampled with sampling interval $T_s = 0.025$ sec and passed through an ideal low-pass filter whose frequency response is shown in the figure

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Signals and Systems

The spectrum of output signal will be ?

(A)

$Y(j\omega)$

π

-40π 0 40π ω

(B)

$Y(j\omega)$

4π

π

-30π 0 30π ω

(C)

$Y(j\omega)$

4π

π

-40π -30π 0 30π 40π ω

(D) Zero

Q.5 Determine the Nyquist sampling rate for the signal $x(t) = \text{sinc}(50\pi t)\text{sinc}(100\pi t)$

(A)50   (B)100   (C)125   (D)150

Answer Key

	1	2	3	4	5	6	7
Type 1	B	C	A	D			
Type 2	D	B	D	C	A		
Type 3	D	A	D	B	C		
Type 4	C	A	A	D			
Type 5	B	A	A	D	C	C	
Type 6	B	B	B	D	A	A	B
Type 7	A	B	B	C	C	A	C
Type 8	B	C	A	D	A		
Type 9	C	A	D	C	B		
Type 10	A	A	C	A	B		
Type 11	D	B	B	B	A		
Type 12	D	D	A	B	A		
Type 13	B	C					
Type 14	A	A	B	D	C		
Type 15	B	D	C	B	C		
Type 16	A	C	D	B			
Type 17	A	C	C	D	C	D	D
Type 18	B	A	B	B	D		

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